Vibrational Analysis Of Laminated Composite Beam

Thesis submitted in partial fulfillment of the requirements for the Degree of

Bachelor of Technology (B. Tech.)
In
Mechanical Engineering

By
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CERTIFICATE

This is to certify that the thesis entitled **," Vibrational Analysis Of Laminated Composite Beam "** submitted by **Nikhil Kumar** has been carried out under my supervision in partial fulfillment of the requirements for the Degree of Bachelor of Technology (B. Tech.) in Mechanical Engineering at National Institute of Technology, NIT Rourkela, and this work has not been submitted elsewhere before for any other academic degree/diploma.

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ACKNOWLEDGEMENT

It is with a feeling of great pleasure that I would like to express my most sincere heartfelt gratitude to Prof. R. K. Behera, Asst. Professor, Dept. of Mechanical Engineering, NIT Rourkela for suggesting the topic for my summer report and for his ready and able guidance throughout the course of my preparing the report. I am greatly indebted to him for his constructive suggestions and criticism from time to time during the course of progress of my work.

I express my sincere thanks to Prof. R.K Sahoo, Head of the Department of Mechanical Engineering, NIT, Rourkela for providing me the necessary facilities in the department. I am also thankful to all the staff members of the Department of Mechanical Engineering.

I feel pleased and privileged to fulfill my parent ambition and I am greatly indebted to them for bearing the inconvenience during my B.E. course.

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ABSTRACT

A composite beam consists of laminate consisting of more than one lamina bonded together through their thickness. Thicknesses of Lamina are of order 0.005 inch (0.125mm), implying that to take realistic loads several laminae will be required. For a typical unidirectional lamina the mechanical properties are severely limited in the transverse direction. Stacking several unidirectional layers, may lead to an optimum laminate for unidirectional loads. However, this would not be desirable for complex loading and stiffness requirements,. One can overcome this problem by making a laminate with layers stacked at different angles to withstand different loading and stiffness requirements. Usually more than one lamina are bonded together through their thickness to get real structure. Each layer can be identified, its material, its angle of orientation with respect to a reference axis and by location in the laminate.

Reduced stiffness matrix was obtained by using properties of composite material. These properties are Longitudinal elastic modulus , Transverse elastic modulus , Major poisons ratio and Shear modulus . Using these properties composite compliance matrix was obtained . Inverse of compliance matrix was taken and reduced stiffness matrix was obtained . Then reduced stiffness matrix for each and every layer was calculated . Reduced stiffness matrix for each and every layer was calculated taking in consideration angles of the fiber in lamina .

Mid plane symmetry was taken and position of each layer was calculated with respect to mid plane .D11 matrix was determine by formula using relative position of layer from mid plane and reduced stiffness matrix of all lamina (effect of angle of fiber was included) . Density of composite material was obtained by using densities of each material and there volume composition. Value of natural frequency in rad/sec and per sec was obtain by using formulation for finding frequency of composite material . Frequency was obtained for all the supports i.e

simple –simple , simple – clamped , clamped –clamped and clamped –free and for first three mode of vibration . Glass/epoxy , graphite /epoxy and boron epoxy composites were used to obtain tabulation for natural frequency in hertz . Comparison of frequency for these composite , frequency of composite under different mode condition were done for these composites and required histogram was plotted . Taking beam as euler beam , equation of euler beam was considered and was solved for simple –simple case taking in consideration boundary condition of simple – simple support condition i.e displacement at support and bending moment at support is equal to zero .

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CHAPTER - 1

INTRODUCTION

For structural applications where high strength-to weight and stiffness-to-weight ratios are required the fiber-reinforced composite materials are ideal . By altering lay-up and fiber orientations composite material can be tailored to meet the particular requirements of stiffness and strength .The ability to manufacture a composite material as per its job is one of the most significant advantages of composite material over an ordinary material .Due to the high strength to low weight ratio , resistance in fatigue and low damping factor , composite materials have wide range of applications in car and aircraft industries. Research in the design of mechanical, aerospace and civil structure and development of composite materials has grown tremendously in few decades . On a structure dynamic loading can vary from recurring cyclic loading of the same repeated magnitude , such as a unbalanced motor which is turning at a specified number of revolutions per minute on a structure (for example), to the other extreme of a short time , intense , nonrecurring load , termed shock or impact loading , such as a bird striking an aircraft component during flight . A continuous infinity of dynamic loads exists between these extremes of harmonic oscillation and impact. associated mode shapes. There are infinity of mode shapes and natural frequency in a continuous structure.One of the bases for designing and modeling of industrial products is finding the free vibration characteristics of Laminated Composite Beam . Beam analysis is important in mechanical and civil structural design such as railways , car suspension system and structural foundation . Free vibration of uniform cross-section LCBs with no foundation have been investigated by many researchs .

CHAPTER-2

LITERATURE REVIEW

Li Jun , Hua Hongxing and Shen Rongying introduced a dynamic finite element technique for free vibration analysis of typically laminated composite beams on the idea of 1st order shear deformation theory. The influences of Poisson impact, couplings among extensional, bending and torsional deformations, shear deformation and rotary inertia are incorporated within the formulation. The dynamic stiffness matrix is formulated primarily based on the precise solutions of the differential equations of motion governing the free vibration of generally laminated composite beam. The effects of Poisson effect, material anisotropy, slender ratio, shear deformation and boundary condition on the natural frequencies of the composite beams are studied thoroughly by specific carefully rigorously selected examples. The numerical results of natural frequencies and mode shapes are presented and, whenever possible, compared to those previously published solutions so as to demonstrate the correctness and accuracy of the current technique [2.1].

R.A. Jafari-Talookolaei and M.T.Ahmadian investigated free vibration analysis of a cross-ply laminated composite beam (LCB) onPasternak foundation . Natural frequencies of beam on Pasternak foundation are computed using finite element technique (FEM) on the idea of Timoshenko beam theory. Impact each shear deformation and rotary inertia are implemented within the modeling of stiffness and mass matrices. The model was designed such that it may be used for single-stepped cross-section, stepped foundation and multi-span beams. Results of few examples are compared with finding in literature and genuine agreements were achieved.

Natural frequencies of LCBs with different layers arrangements symmetric are compared. For multi-span beam, variation of frequency with respect to range of spans was conjointly studied.[2.2].

Mesut Simsek and Turgut Kocaturk analyzed free vibration of beams with totally different boundary conditions among the framework of the third-order shear deformation theory. The boundary conditions of beams are satisfied using Lagrange multipliers. To use the Lagrange's equations, trial functions denoting the deflections and therefore the rotations of the cross-section of the beam are expressed in polynomial type. Using Lagrange's equations, the problem is reduced to the answer of a system of algebraic equations. the primary six eigenvalues of the thought of beams are calculated for various thickness-to-length ratios. The results are compared with the previous results primarily based on Timoshenko and Euler–Bernoulli beam theories.[2.3]

Jaehong Lee presented Free vibration analysis of a laminated beam with delamination , a layerwise theory. Equations of motion are derived from the Hamilton's principle, and a finite technique is developed to formulate the problem. Numerical results are obtained and compared with those of alternative theories addressing the consequences of the lamination angle, location, size and range of delamination on vibration frequencies of delaminated beams. It's}found that a layer wise approach is adequate for vibration analysis of delaminated composites.[2.4]

Patel have investigated Non-linear free flexural vibration/post buckling analysis of laminated orthotropic beams/columns on a 2 parameter elastic foundation (Pasternak). They have used Von-Karman strain-displacement relations and formulation consisted effects of shear deformation and rotary inertia.[2.5].

Thambiratnam and Zhug have implemented finite element technique to review the free vibration analysis of isotropic beams with uniform cross section on an elastic foundation using Euler-Bernoulli beam theory.[2.6]

Banerjee has investigated the free vibration of axially laminated composite Timoshenko beams using dynamic stiffness matrix technique.[2.7]

Subramanian has investigated free vibration analysis of LCBs by using 2 higher order displacement primarily based on shear deformation theories and finite elements. Each theories assume a quintic and quartic variation of in-plane and transverse displacements within the thickness coordinates of the beam respectively. Results indicate application of those theories and finite element model leads to natural frequencies with higher accuracy.[2.8].

CHAPTER -3

LAMINATE CODE

A composite beam consists of laminate consisting of more than one lamina bonded together through their thickness .Lamina thicknesses are of order 0.125mm , as a result of which several lamina will be required to take real load. For a typical unidirectional lamina mechanical properties are severely limited in the transverse direction. After stacking several unidirectional layer we can get optimum laminate for unidirectional loads . This is not desirable for complex stiffness and loading requirements. By stacking laminate layers at different angles for required stiffness and loading this problems can be solved .

Many laminas are bonded together through thickness in real laminated structure . Each lamina can be distinguished by its material , position in laminate and orientation of fiber with respect to reference axis . Angle of ply represents each lamina and slash sign separates it from other plies .

Special notations are used for symmetric laminates, laminates with adjacent lamina of the same orientation or of opposite angles, and hybrid laminates. The following examples illustrate the laminate code.

0
–45
90
60
30

[0/–45/90/60/30] denotes the code for the above laminate. It consists of five plies, each of which has a different angle to the reference *x*-axis. A slash separates each lamina. The code also implies that each ply is made of the same material and is of the same thickness. Sometimes, [0/–45/90/60/30]*T* may also denote this laminate, where the subscript *T* stands for a total laminate.

0
–45
90
90
60
0

[0/–45/90^2/60/0] denotes the laminate above, which consists of six plies.Because two 90° plies are adjacent to each other, 90^2 denotes them, where the subscript 2 is the number of adjacent plies of the same angle.

0
–45
60
60
–45
0

[0/–45/60]*s* denotes the laminate above consisting of six plies. The plies above the midplane are of the same orientation, material, and thickness as the plies below the midplane, so this is a symmetric laminate. The top three plies are written in the code, and the subscript *s* outside the brackets represents that the three plies are repeated in the reverse order.

0
–45
60
–45
0

[0/–45/$\overline{60}$]s denotes this laminate, which consists of five plies. The number of plies is odd and symmetry exists at the midsurface; therefore, the 60 ° ply is denoted with a bar on the top.

Graphite /epoxy
Boron/epoxy
Boron/epoxy
Boron/epoxy
Boron/epoxy
Graphite/epoxy

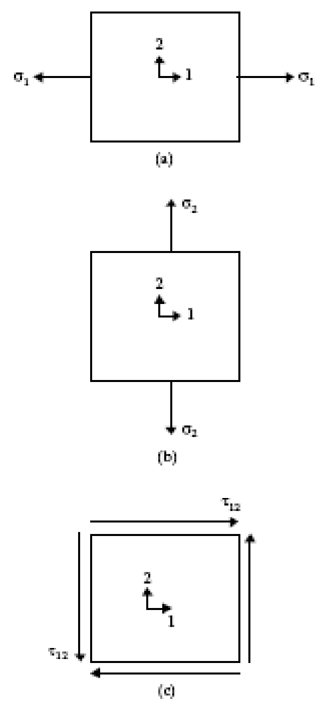
0
45
–45
–45
45
0

[0*Gr*/±45*B*]*s* denotes the above laminate. It consists of six plies; the 0 ° plies are made of graphite/epoxy and the ±45°angle plies are made of boron/epoxy. Note the symmetry of the laminate. Also, the ±45° notation indicates that the 0° ply should be followed by a +45 ° angle ply and then by a –45 ° angle ply. A notation of ± 45 ° would indicate the –45° angle ply is followed by a +45 ° angle ply. [3.1]

CHAPTER-4

DETERMINATION OF REDUCED STIFFNESS MATRIX FOR UNIDIRECTIONAL LAMINA

To determine the reduced stiffness matrix Longitudinal elastic modulas , Transverse elastic modulas , Major poisons ratio , Minor poison ratio and Shear modulas are required for the given composite . Formula used for determining reduced stiffness matrix are following .



Application of stresses to find engineering constants of a unidirectional lamina.

$$Q_{11} = \frac{E_1}{1 - \nu_{21}\nu_{12}}, \qquad [4.1]$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{21}\nu_{12}}, \quad [4.2]$$

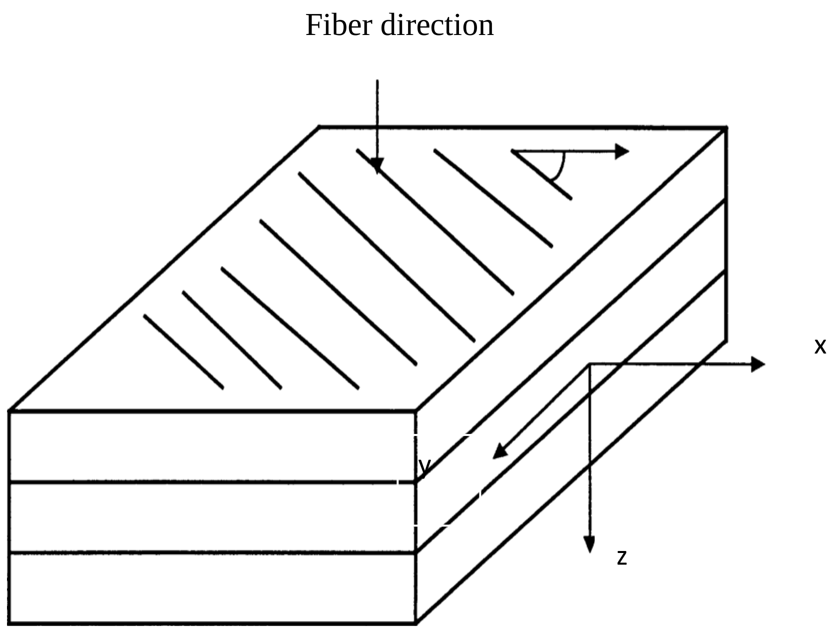
$$Q_{22} = \frac{E_2}{1 - \nu_{21}\nu_{12}}, \text{ and} \quad [4.3]$$

$$Q_{66} = G_{12}. \quad [4.4]$$

Since normal stresses applied in the 1–2 direction do not result in any shearing strains in the 1–2 plane because *Q*16 = *Q*26 = 0 = *S*16 = *S*26 therefore unidirectional lamina is a *specially orthotropic* lamina . Also, the shearing stresses applied in the 1–2 plane do not result in any normal strains in the 1 and 2 directions because *Q*16 = *Q*26 = 0 = *S*16 = *S*26. A woven composite with its weaves perpendicular to each other and short fiber composites with fibers arranged perpendicularly to each other or aligned in one direction also are *specially orthotropic*.

CHAPTER- 5
DETERMINATIONAL OF D11 MATRIX
5.1Hooke's Law for a Two-Dimensional Angle Lamina

In most laminate , some lamina are placed at an angle because laminate consisting of unidirectional laminae have low stiffness and strength properties in the transverse direction .



Coordinate System

In the 1–2 coordinate system the axes are called the local axes or the material axes. The direction 1 is parallel to the fibers and the direction 2 is perpendicular to the fibers. In some literature, direction 1 is also called the longitudinal direction *L* and the direction 2 is called the transverse direction *T*. The axes in the *x*–*y* coordinate system are called the global axes or the off-axes. The angle between the two axes is denoted by an angle . The stress–strain relationship in the 1–2 coordinate system

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}, \qquad [5.1]$$

where [*T*] is called the transformation matrix and is defined as

$$[T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}, \qquad [5.2]$$

And

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}, \qquad [5.3]$$

$$c = \mathrm{Cos}\,(\theta),$$

$$s = \mathrm{Sin}\,(\theta).$$

Using the stress–strain Equation in the local axes, Equation 5.1 can be written as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1}[Q] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}. \qquad [5.4]$$

The global and local strains are also related through the transformation matrix

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} - [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{bmatrix}, \qquad [5.5]$$

which can be rewritten as

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \qquad [5.6]$$

where [*R*] is the Reuter matrix3 and is defined as

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \qquad [5.7]$$

Then, substituting Equation 5.7 in Equation 5.5 gives

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1}[Q][R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}. \quad [5.8]$$

On carrying the multiplication of the first five matrices on the right-hand side of Equation 5.8

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \quad [5.9]$$

$$\bar{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2,$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^2),$$

$$\bar{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2,$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})s^3c,$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s,$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4).$$

5.2 Strain and Stress in a Laminate

If the strains are known at any point along the thickness of the laminate, the stress–strain Equation 5.9 calculates the global stresses in each lamina:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \qquad [5.10]$$

Laminate strain can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}. \qquad [5.11]$$

The reduced transformed stiffness matrix, corresponds to that of the ply located at the point along the thickness of the laminate. Substituting equation 5.10 in 5.11 .

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}. \qquad [5.12]$$

From Equation 5.12 the stresses vary linearly only through the thickness of each lamina (Figure f1). The stresses, however, may jump from lamina to lamina because the transformed reduced-stiffness matrix changes from ply to ply because depends on the material and orientation of the ply.

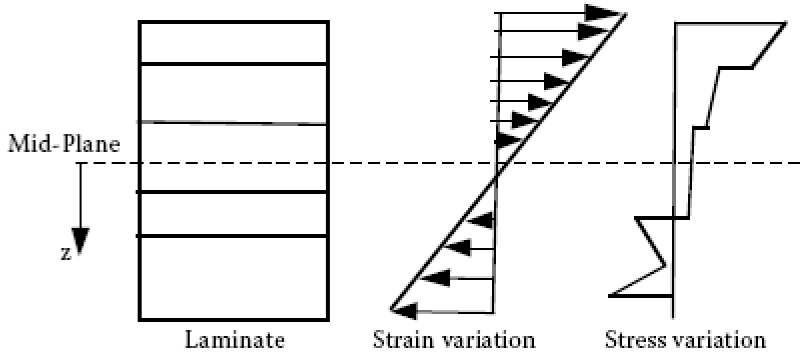


Fig:f1

5.3 Strain and Stress Variation Through The Thickness Of The Laminate.

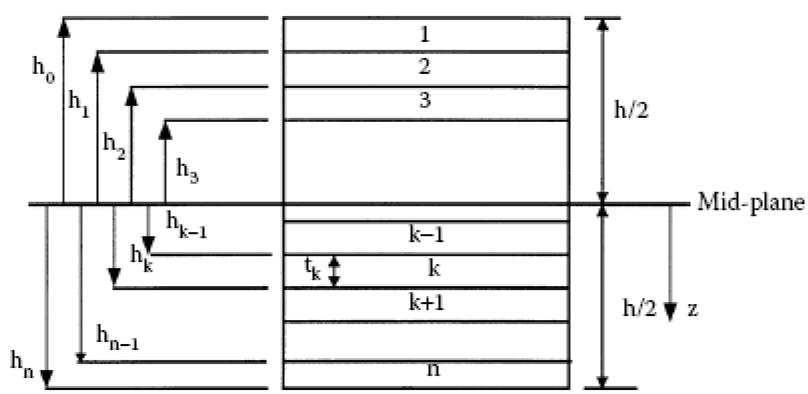


Fig:f2

Coordinate locations of plies in a laminate.

Force and Moment Resultants Related to Midplane Strains and Curvatures

The midplane strains and plate curvatures in Equation 5.11 are the unknowns for finding the lamina strains and stresses. However, Equation 5.12 gives the stresses in each lamina in terms of these unknowns. The stresses in each lamina can be integrated through the laminate thickness to give resultant forces and moments (or applied forces and moments). The forces and moments applied to a laminate will be known, so the midplane strains and plate curvatures can then be found. This relationship between the applied loads and the midplane strains and curvatures is developed in this section. Consider a laminate made of *n* plies shown in Figure F2. Each ply has a thickness of *tk*. Then the thickness of the laminate *h* is

$$h = \sum_{k=1}^{n} t_k .$$

Then, the location of the midplane is *h*/2 from the top or the bottom surface of the laminate. The *z*-coordinate of each ply *k* surface (top and bottom) is given by

$$h_0 = -\frac{h}{2} \ (top\ \ surface),$$

Ply 1: $$h_1 = -\frac{h}{2} + t_1 \ (bottom\ \ surface) .$$

Ply k: (k = 2, 3,…n – 2, n – 1):

$$h_{k-1} = -\frac{h}{2} + \sum_{=1}^{k-1} t \ \ (top\ \ surface)$$

$$h_k = -\frac{h}{2} + \sum_{=1}^{k} t \ \ (bottom\ \ surface) .$$

Ply n:

$$h_{n-1} = \frac{h}{2} - t_n \ (top\ \ surface)$$

$$h_n = \frac{h}{2} \ (bottom\ \ surface) .$$

Integrating the global stresses in each lamina gives the resultant forces per unit length in the *x*–*y* plane through the laminate thickness as

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz,$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y dz,$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} dz, ,$$

where $h/2$ is the half thickness of the laminate.

Similarly, integrating the global stresses in each lamina gives the resulting moments per unit length in the *x*–*y* plane through the laminate thickness as

$$M_x = \int_{-h/2}^{h/2} \sigma_x z dz,$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y z dz,$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z dz,$$

where

Nx, *Ny* = normal force per unit length

Nxy = shear force per unit length

Mx, *My* = bending moments per unit length

Mxy = twisting moments per unit length

These equations can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz,$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} zdz \ ,$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k dz,$$

[5.13]

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k zdz.$$

Substituting equation 5.13 in 5.12 we get

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} dz$$

$$+ \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} z dz$$

And

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} z dz$$

$$+ \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} z^2 dz \ .$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} dz \right\} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \qquad [5.14a]$$

$$+ \left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} z dz \right\} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} z dz \right\} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \qquad [5.14b]$$

$$+ \left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} z^2 dz \right\} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}.$$

Knowing that

$$\int_{h_{k-1}}^{h_k} dz = (h_k - h_{k-1}),$$

$$\int_{h_{k-1}}^{h_k} z dz = \frac{1}{2}(h_k^2 - h_{k-1}^2),$$

$$\int_{h_{k-1}}^{h_k} z^2 dz = \frac{1}{3}(h_k^3 - h_{k-1}^3),$$

and substituting in Equation 5.14 gives

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}, \tag{5.15}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}, \tag{5.16}$$

where

$$A_{ij} = \sum_{k=1}^{n} [(\bar{Q}_{ij})]_k (h_k - h_{k-1}), \quad i = 1,2,6; \quad j = 1,2,6, \tag{5.17}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [(\bar{Q}_{ij})]_k (h_k^2 - h_{k-1}^2), \quad i = 1,2,6; \quad j = 1,\mathbf{2},6, \tag{5.18}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} [(\bar{Q}_{ij})]_k (h_k^3 - h_{k-1}^3), \quad i = 1,2,6; \quad j = 1,\mathbf{2},6. \tag{5.19}$$

CHAPTER-6

VIBRATION OF COMPOSITE BEAM

On a structure dynamic loading can vary from recurring cyclic loading of the same repeated magnitude , such as a unbalanced motor which is turning at a specified number of revolutions per minute on a structure (for example), to the other extreme of a short time , intense , nonrecurring load , termed shock or impact loading , such as a bird striking an aircraft component during flight . A continuous infinity of dynamic loads exists between these extremes of harmonic oscillation and impact. associated mode shapes. Mathematically ,there are infinity of natural frequencies and mode shapes in a continuous structure.

Dynamic loading can vary from intense , nonrecurring load known as shock load such as bird striking aero plane to recurring cyclic loading of magnitude which repeats itself such as unbalanced motors rotating at particular R.P.M . Any structures amplitude may rapidly grows with time if its frequency of oscillation matches its natural frequency .

Structure can be overstressed which leads to its failure or due to large oscillations amplitude may be limited at large value which further leads to fatigue damages.

Time dependent loading should be compared with natural frequency to ensure structural integrity of any structure . These two frequencies should be considerably different . While designing structure over deflecting and overstressing should be taken care of and resonances should be avoided .

ω_n is the natural circular frequency in radians per unit time for the *n*th vibrational mode.

Note that in this case there is one natural frequency for each natural mode shape, for n = 1, 2, 3,..., etc.

ω_n can be expressed as

$$\omega_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{bD_{11}}{\rho_m A}}$$

Transverse-shear effect is not taken into consideration in this equation .

For each n there would be different natural frequency .

ω_n can be written as

$$\omega_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{bD_{11}}{\rho_m A}}$$

Where b= Breadth of beam

A= Area of cross section

L= Length of beam

ρ_m = Density of composite material .

α^2 = Constant

Value of α^2 is given in following table for all types of support i.e simple-simple , clamped – clamped , simple – clamped and clamped – free .

Frequency in hertz can be determined by $f_n = \omega_n / 2\pi$. The values α^2 of have been catalogued by Warburton , Young and Felgar and Felgar . The natural frequencies of a free-free supported beam is equal to natural frequency of clamped-clamped supported beam. Natural frequencies would be lower if tansverse shear deformation effects were included .

Tabulation of α^2 Values for Use in Determining Beam-Natural Frequencies

n	Simple-Simple	Simple-Clamped	Clamped-Clamped	Clamped-Free
1	9.870	15.42	22.37	3.516
2	39.48	49.96	61.67	22.03
3	88.83	104.25	120.90	61.70

Density of composites

Density of composite depends upon its constituent materials . It depends upon density and volume fraction of constituent material.

Density of composite = Density of first material * Volume fraction of first material + Density of second material * Volume fraction of second material

CHAPTER-7

RESULT AND DISCUSSION

Tabulation for frequency

Let us take assumption of physical structure of beam for calculation purpose .

Length =L=0.1179 m

Breadth = b=12.7*10^{-3}m

Height =h=3.38 *10^{-3} m

A=Area of cross section =4.29*10^{-5} m^2

Tabulation for Boron/ epoxy composite [0^0/30^0/-45]

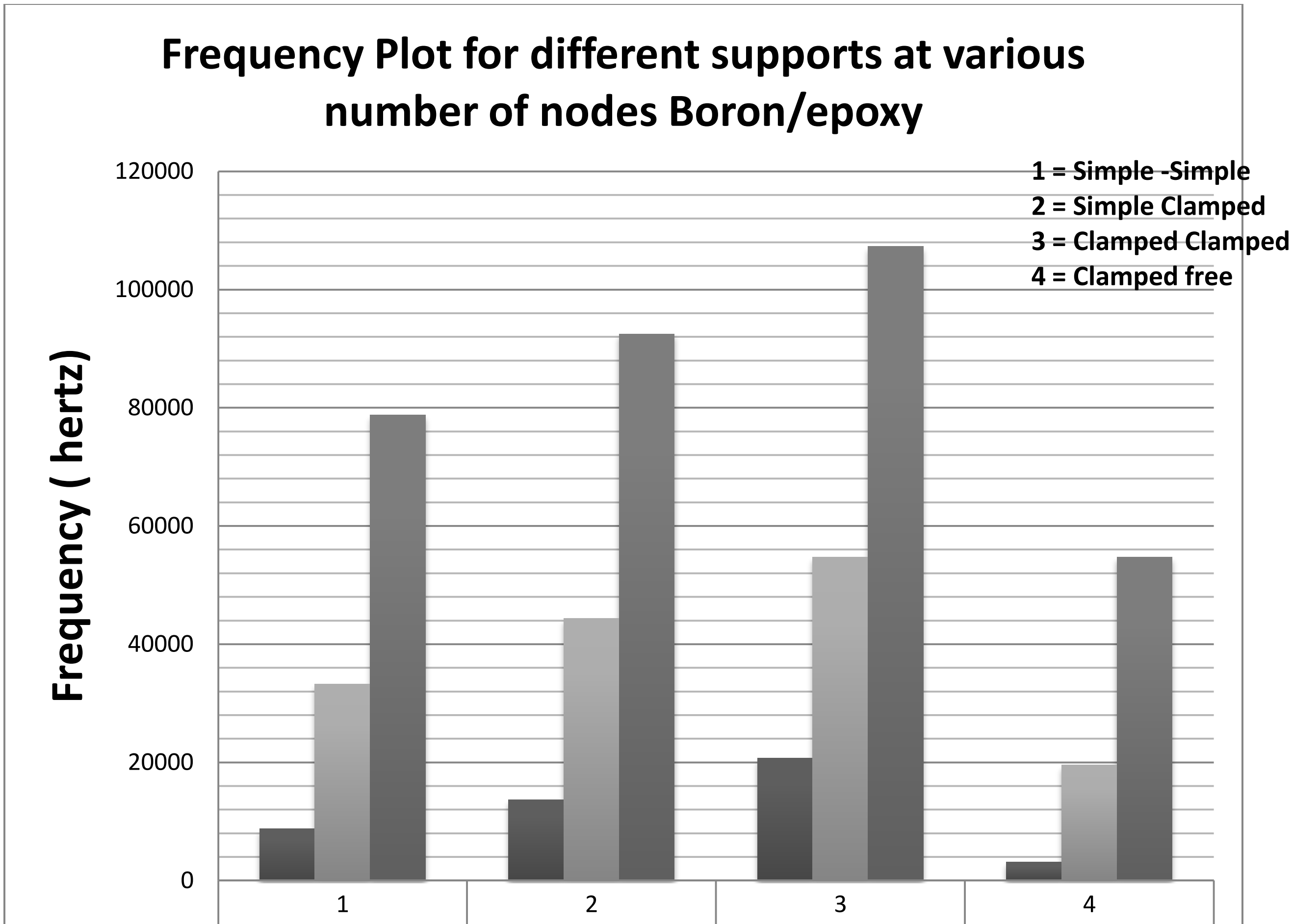
E_1= Longitudinal elastic modulas = 204GPa

E_2=Transverse elastic modulas = 18.5 GPa

$V_{12=}$Major poisons ratio =0.23

G_{12}=Shear modulas =5.59 GPa

	freq. n=1(hertz)	freq. n=2 (hertz)	freq. n=3 (hertz)
Simple -Simple	8758.62	33259.37	78826.85
simple clamped	13683.55	44334	92510.4
clamped clamped	20738.3	54725.34	107285.45
clamped free	3120.06	19549.2	54751.96



	1	2	3	4
freq. n=1	8758.62	13683.55	20738.3	3120.06
freq. n=2	33259.37	44334	54725.34	19549.2
freq. n=3	78826.85	92510.4	107285.45	54751.96

Tabulation for graphite/epoxy composite [$0^0/30^0/-45$]

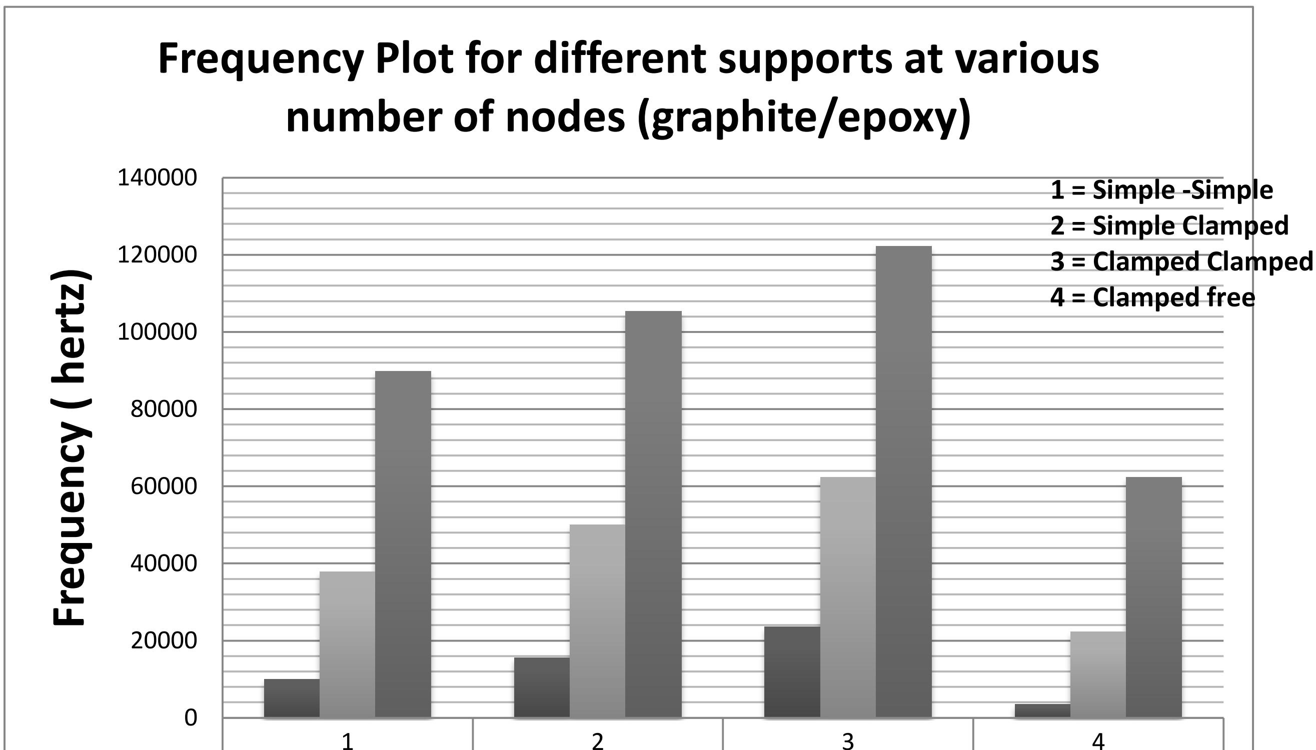
E_1= Longitudinal elastic modulas = 181GPa

E_2=Transverse elastic modulas = 10.3 GPa

$V_{12=}$Major poisons ratio =0.28

G_{12}=Shear modulas =7.17 GPa

	freq. n=1(hertz)	freq. n=2 (hertz)	freq. n=3 (hertz)
Simple -Simple	9976.8	37885.5	89791.22
simple clamped	15586.85	49995.2	105378.08
clamped clamped	23622.88	62337.32	122208.24
clamped free	3554.04	22268.38	62367.64



	1	2	3	4
freq. n=1	9976.8	15586.85	23622.88	3554.04
freq. n=2	37885.5	49995.2	62337.32	22268.38
freq. n=3	89791.22	105378.08	122208.24	62367.64

Tabulation for glass/epoxy composite [$0^0/30^0/-45$]

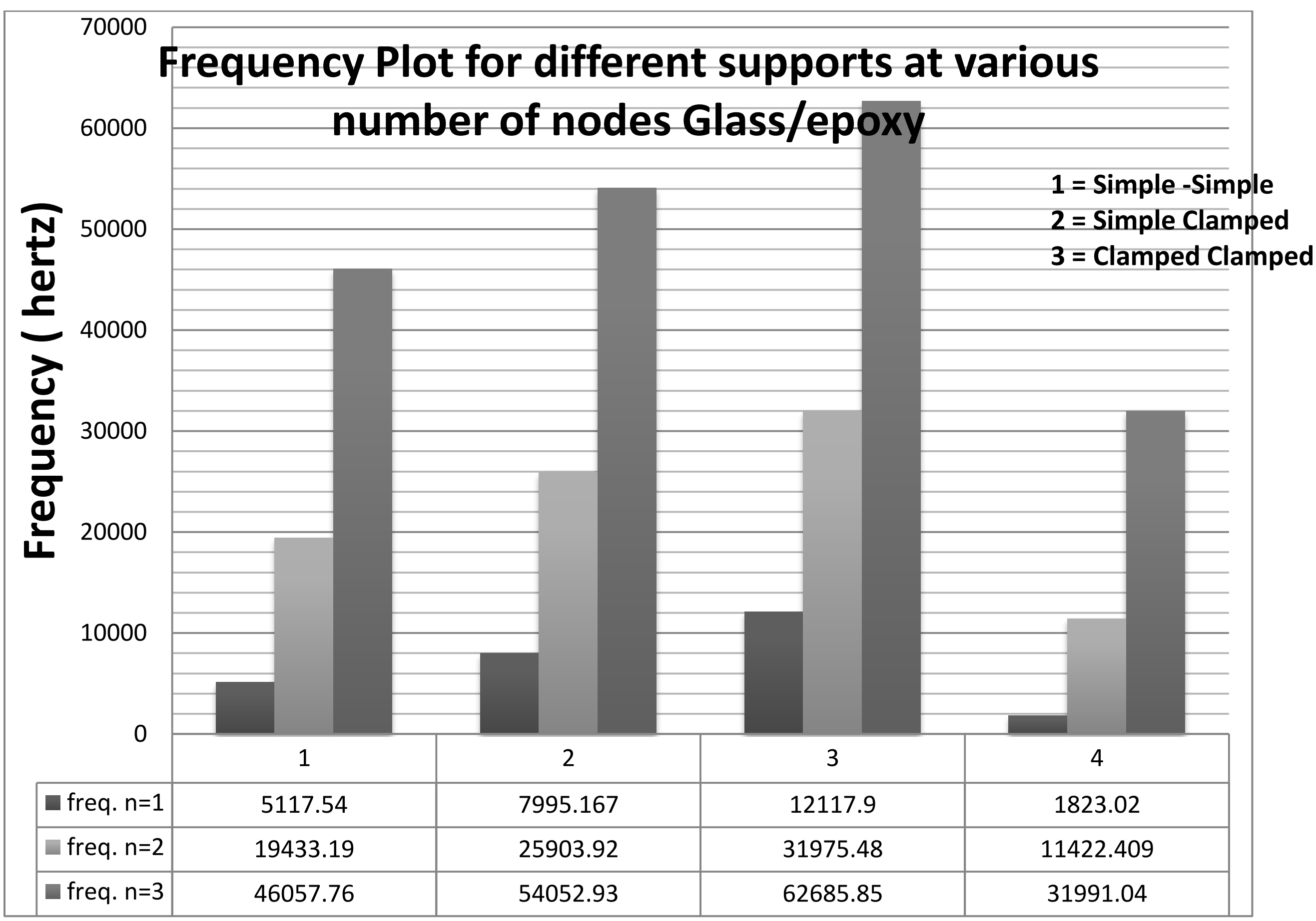
E_1= Longitudinal elastic modulas =38.6 GPa

E_2=Transverse elastic modulas = 8.27 GPa

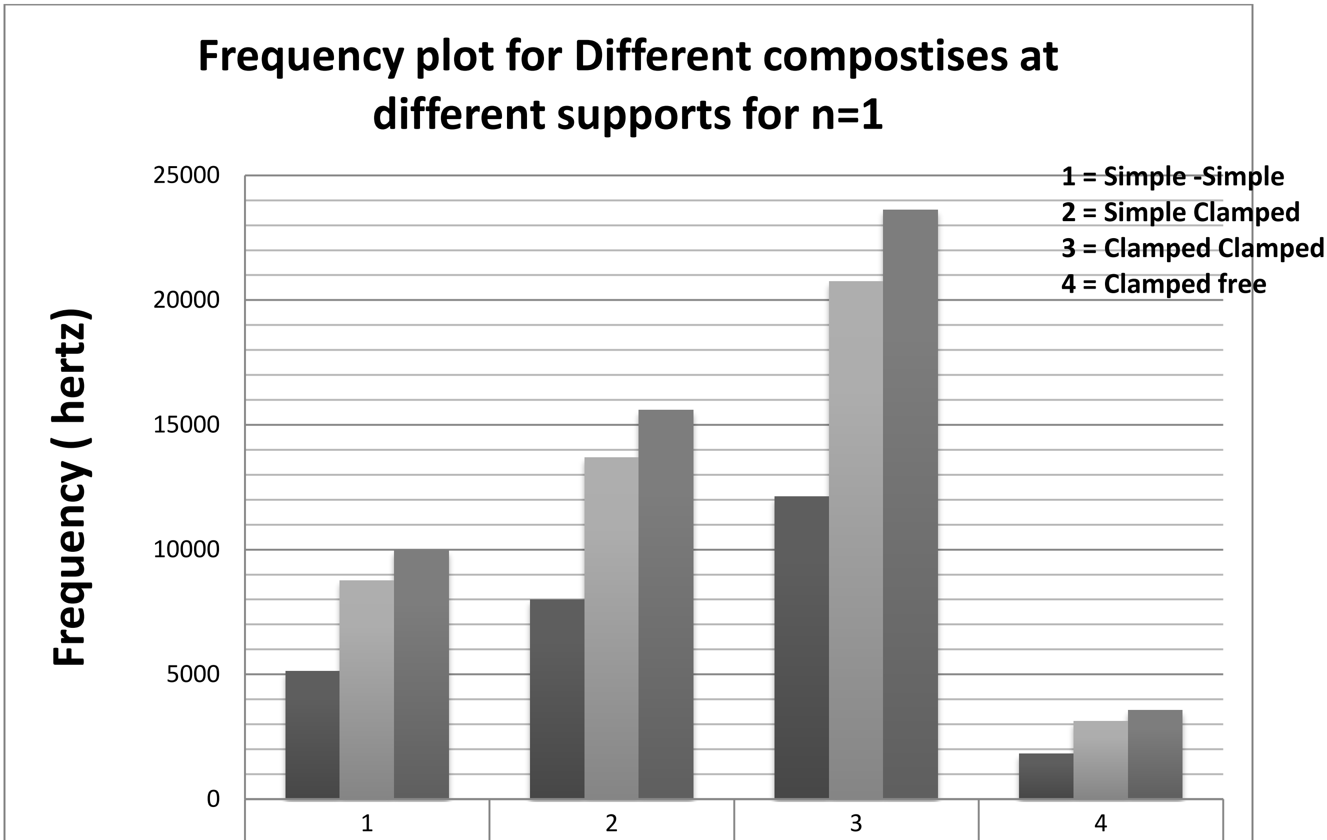
$V_{12=}$Major poisons ratio =0.26

G_{12}=Shear modulas =4.14 GPa

	freq. n=1(hertz)	freq. n=2 (hertz)	freq. n=3 (hertz)
Simple -Simple	5117.54	19433.19	46057.76
simple clamped	7995.167	25903.92	54052.93
clamped clamped	12117.9	31975.48	62685.85
clamped free	1823.02	11422.409	31991.04



Comparison of Frequencies of Different Composites at Different Supports at n=1



	1	2	3	4
Glass/epoxy	5117.54	7995.167	12117.9	1823.02
boron/epoxy	8758.62	13683.55	20738.3	3120.06
Grphite/epoxy	9976.8	15586.85	23622.88	3554.04

CHAPTER -8

MODE SHAPE

Euler–Bernoulli beam theory provides a means of calculating the load-carrying and deflection characteristics of beams. It is also known as classical beam theory, engineer's beam theory or just beam theory. This is valid for beam undergoing small deflection and subjected to lateral loads only. It is applied on thick beams and is a special case of TBH which account for shear deformation.

For mode shape calculation Euler beam was considered .

For Euler beam y(x,t)=U(x)*T(t)

y= Deflection in transverse didection .

T(t)= Time dependent function .

U(x) = Function depending upon distance from support in x direction .

Solving U(x) we get $\bar{v}(x)$. For constant time y when is responsible for mode shape is fucntion of x only .

Solution of U(x) is

$$\bar{v}(x) = A_1 \sin \beta x + A_2 \cos \beta x + A_3 \sinh \beta x + A_4 \cosh \beta x$$

End Condition	General Equation for mode shape
Simple - Simple	$\bar{v}(x)$ =Cnsin(βnx)
Simple - Clamped	$\bar{v}(x)$ =Cn sin(βnx)-sinh(βnx)+((sin(βnx)-sinh(βnx))/(cos(βnx)-cosh(βnx))(cosh(βnx)-cos(βnx))
Clamped - Clamped	$\bar{v}(x)$ =Cn sinh(βnx)-sin(βnx)+((sinh(βnx)-sin(βnx))/(cos(βnx)-cosh(βnx))(cos(βnx)-cosh(βnx))
Clamped - Free	$\bar{v}(x)$ =Cn sin(βnx)-sinh(βnx)-((sin(βnx)+sinh(βnx))/(cos(βnx)+cosh(βnx))(cos(βnx)-cosh(βnx))

Solution of $\bar{v}(x)$ can be obtained by using boundary conditions :

Boundary Conditions :

- For free ends bending moment and shear force is equal to zero .
- For simply supported end deflection and bending moment is equal to zero .
- For clamped end deflection and slope is equal to zero .

Mode plotting

In order to get plot of mode shape of vibration of beam $\bar{v}(x)$ equation is used and required data is taken from Table 9.2 (Dynamic of vibration bt Magd Abdel Wahab) .

Equations for plotting

simple - simple

y=sin(26.6462x);

y=sin(53.2924x);

y=sin(79.9387x);

Simple - clamped

y=sin(33.2994x)-sinh(33.2994x)+((sin(33.2994x)-sinh(33.2994x))/(cos(33.2994x)-cosh(33.2994x))(cosh(33.2994x)-cos(33.2994x));

y=sin(59.9491x)-sinh(59.9491x)+((sin(59.9491x)-sinh(59.9491x))/(cos(59.9491x)-cosh(59.9491x))(cosh(59.9491x)-cos(59.9491x));

y=sin(86.5988x)-sinh(86.5988x)+((sin(86.5988x)-sinh(86.5988x))/(cos(86.5988x)-cosh(86.5988x))(cosh(86.5988x)-cos(86.5988x));

clamped - clamped

y=sinh(40.1187x)-sin(40.1187x)+((sinh(40.1187x)-sin(40.1187x))/(cos(40.1187x)-cosh(40.1187x))(cos(40.1187x)-cosh(40.1187x));

y=sinh(66.6070x)-sin(66.6070x)+((sinh(66.6070x)-sin(66.6070x))/(cos(66.6070x)-cosh(66.6070x))(cos(66.6070x)-cosh(66.6070x));

y=sinh(93.2567x)-sin(93.2567x)+((sinh(93.2567x)-sin(93.2567x))/(cos(93.2567x)-cosh(93.2567x))(cos(93.2567x)-cosh(93.2567x));

clamped - free

y=sin(15.9033x)-sinh(15.9033x)-((sin(15.9033x)+sinh(15.9033x))/(cos(15.9033x)+cosh(15.9033x))(cos(15.9033x)-cosh(15.9033x));

y=sin(39.8134x)-sinh(39.8134x)-((sin(39.8134x)+sinh(39.8134x))/(cos(39.8134x)+cosh(39.8134x))(cos(39.8134x)-cosh(39.8134x));

y=sin(66.6157x)-sinh(66.6157x)-((sin(66.6157x)+sinh(66.6157x))/(cos(66.6157x)+cosh(66.6157x))(cos(66.6157x)-cosh(66.6157x));

Matlab coding for simple – simple support mode generation .

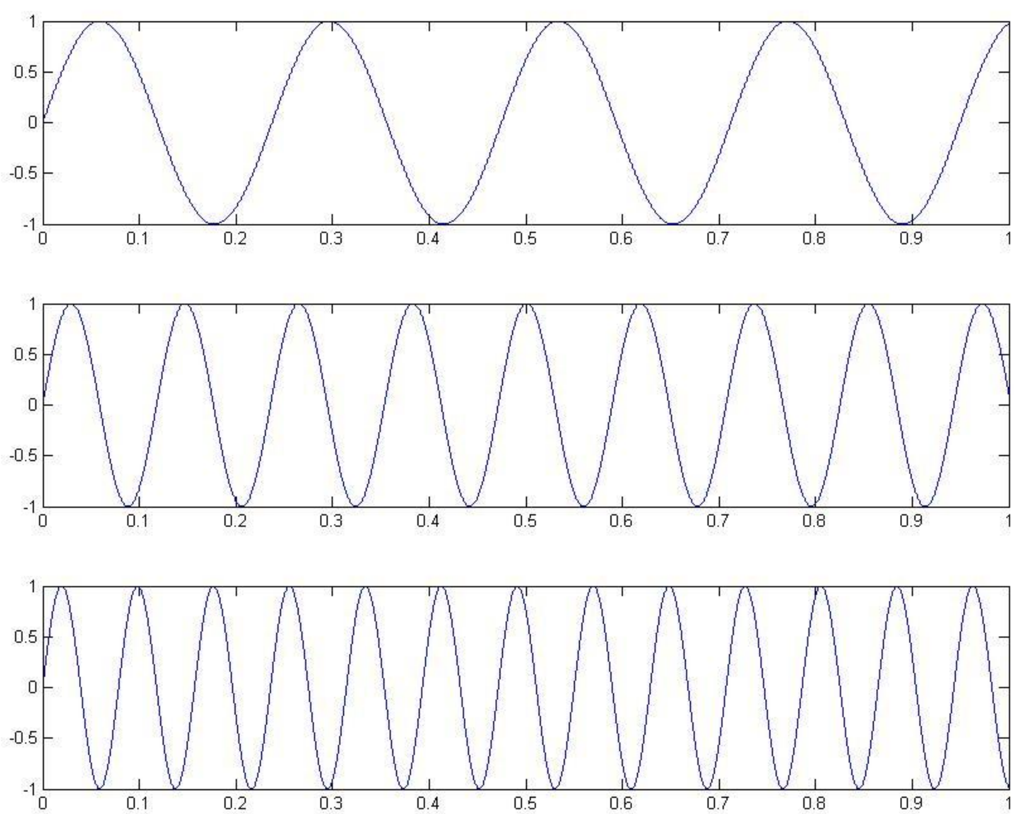
x1=0:0.00001:1

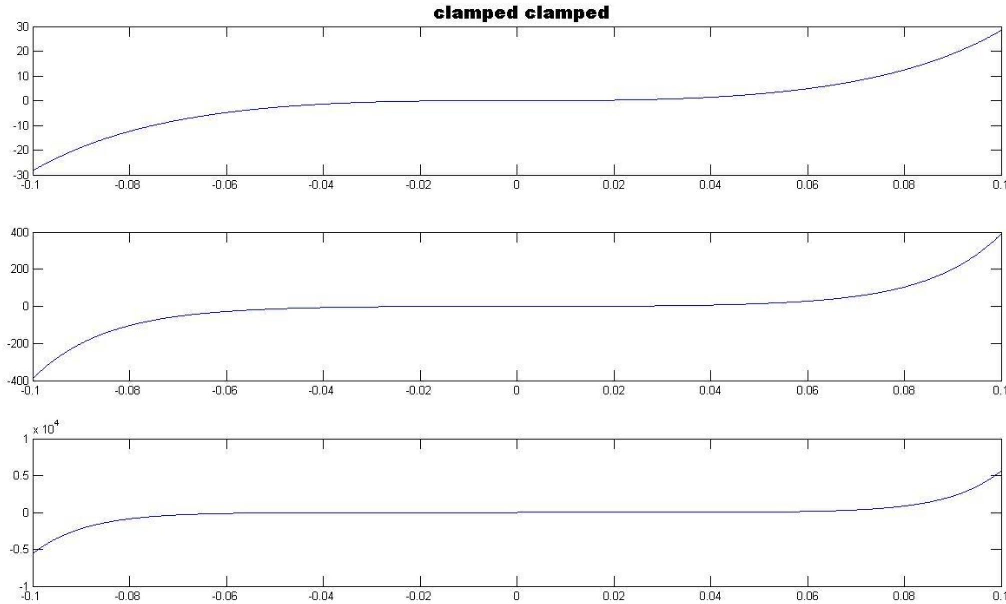
y1=sin(26.462*x1)

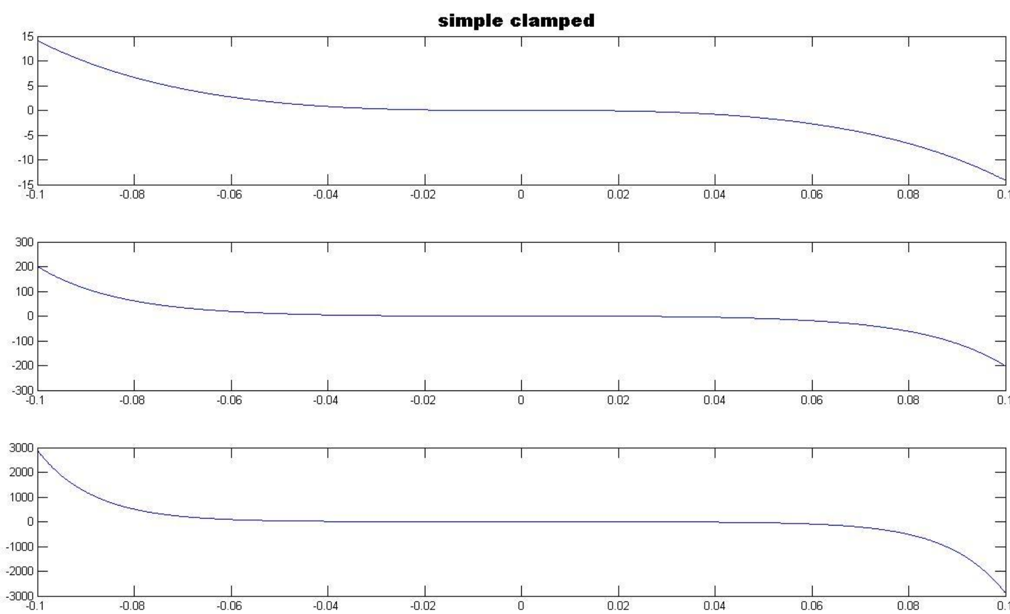
y2=sin(53.2924*x1)

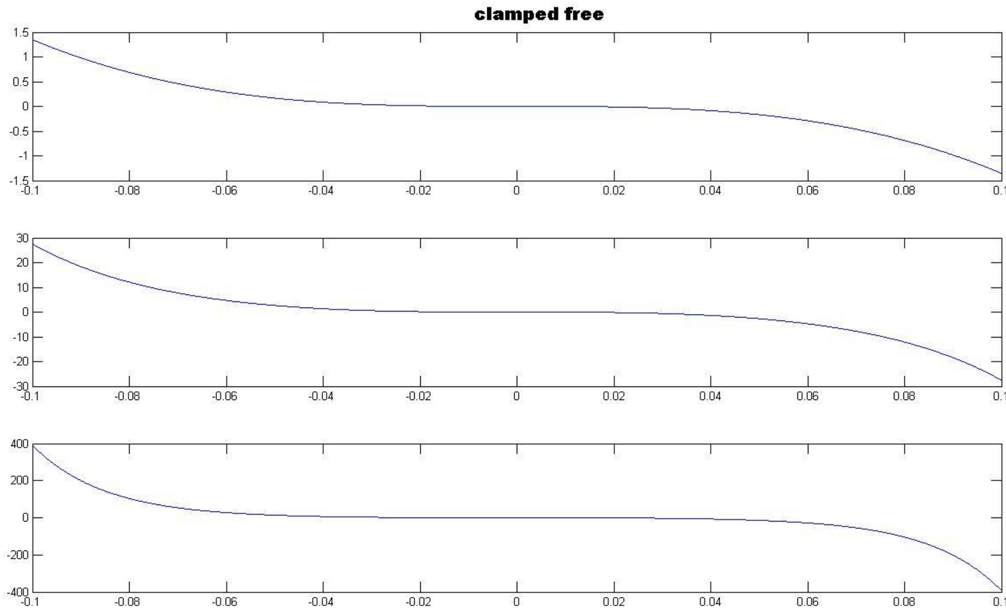
```
y3=sin(79.9387*x1)

subplot(3,1,1);plot(x1,y1)

subplot(3,1,2);plot(x1,y2)

subplot(3,1,3);plot(x1,y3)
```

Clamped –Clamped




clamped clamped
30
20
10
0
-10
-20
-30
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
400
200
0
-200
-400
x 10^4
1
0.5
0
-0.5
-1


simple clamped
15
10
5
0
-5
-10
-15
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
300
200
100
0
-100
-200
-300
3000
2000
1000
0
-1000
-2000
-3000


clamped free
1.5
1
0.5
0
-0.5
-1
-1.5
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
30
20
10
0
-10
-20
-30
400
200
0
-200
-400

CHAPTER -9

VIBRATIONAL ANALYSIS ON ANSYS

Vibrational analysis of a beam can be done on ansys by providing structural data and load conditions on different supports. Structural data which are required for simple vibrational analysis of beam on ansys are

- Young's modulus
- Poisson's ratio
- Density
- Length , Breadth and Height .

Vibrational analysis of graphite/ epoxy composite beam for clamped-free condition :

For the sake of simplicity laminate is considered to be of unidirectional lamina and an additional force of 1000N is provided at middle of beam during analysis to make deflection visible .

Longitudinal elastic modulas =181 GPa

Transverse elastic modulas =10.3 GPa

Major poisons modulas =0.28

Length of beam = 0.11m

Breadth of beam = $12.7*10^{-3}$m

Height of beam =$3.38 *10^{-3}$ m

Area of cross section =$4.29*10^{-5}$ m^2

Area moment of inertia = $bh^3/12$ =$4.08*10^{-11}$ m^4

Procedure for vibrational analysis of beam on ansys :

SELECTION OF ELEMENT TYPE

1. Main Menu—Preprocessor--Element Type--Add/Edit/Delete

2. In the table Library of Element Types select beam and then select 2D elastic 3.

REAL CONSTANTS FOR BEAM ELEMENT

· Main Menu—Preprocessor--Real Constants--Add/Edit/Delete→ Add

· Input the following values in Cross-sectional area=4.29e−5;Area moment of inertia=4.08e−11; [E] Total beam height=3.38e-3. Click OK button to close the window, after inputting these values.

PROPERTIES OF MATERIAL

1. Main Menu—Preprocessor--Material Properties--Material Models.

2.Define Material Model Behavior

3.Structural—Linear—Elastic--Isotropic.

4.Linear Isotropic Properties for Material 1--Input Young’s modulus of 181e9 to EX (box) and Poisson ratio of 0.28 to PRXY (box).

5.Density--Density for Material 1--Input the value of density, 1570 to DENS(box).

CREATE KEYPOINTS

1. Main Menu—Preprocessor—Modeling--Create Key points--In Active CS .

2. Click Apply after inputting 1 to NPT Key Point number box [0,0,0] to [X, Y, Z] Location in active CS box . In the same window, input 2 to NPT Key point number box [0.11, 0,0] to Location in active CS box.

CREATING A LINE FOR BEAM ELEMENT

1.Main Menu—Preprocessor—Modeling—Create—Lines—Lines--Straight Line

2.Create Straight Line and pick the keypoints 1 and 2 .

CREATING MESH IN A LINE

1. Main Menu—Preprocessor—Meshing--Size Controls--Manual Size—Lines--All Lines

2.In window Element Sizes on All Selected Lines--Input the number of 10 to NDIV box. This means that a line is divided into 10 elements.

3. Main Menu—Preprocessor—Meshing—Mesh—Lines.

4.In the window Mesh Lines--Click the line shown in Graphics of ANSYS .

BOUNDARY CONDITIONS

1. Main Menu—Solution--Define Loads--Apply –Structural—Displacement--On Nodes.

2. On Nodes apply U,ROT, click OK after picking the node at the left.

· In order to set the boundary condition, select UX and UY in the box Lab2. In the box VALUE input 0 and, then, click OK.

Appling force :

Solution -Loads- Apply--Structural- Force/Moment--On Keypoints.
Pick keypoint at middle of beam , then "OK" in the picking menu, choose "FY" for "Lab", and enter -1000
for the force value. Click on "OK".

Execution of the analysis

DEFINITION OF THE TYPE OF ANALYSIS

1. ANSYS Main Menu—Solution--Analysis Type--New Analysis

2.New Analysis--Modal

3. Main Menu—Solution--Analysis Type--Analysis Options

4. Modal Analysis--Subspace of MODOPT--input 3 in the box of No. of modes to extract

5.Subspace Modal Analysis--Input 10000 in the box of FREQE

EXECUTION OF CALCULATION

1. ANSYS Main Menu—Solution--Solve-- Current LS

2.Solve Current Load Step--STATUS Command.

POST PROCESSING

READING OF THE CALCULATED RESULTS OF THE FIRST MODE OF VIBRATION

1. Main Menu--General Postproc--Read Results--First Set

PLOT THE CALCULATED RESULTS

1.ANSYS Main Menu--General Postproc--Plot Results-- Deformed Shape

2.Plot Deformed Shape--Def+Undeformed

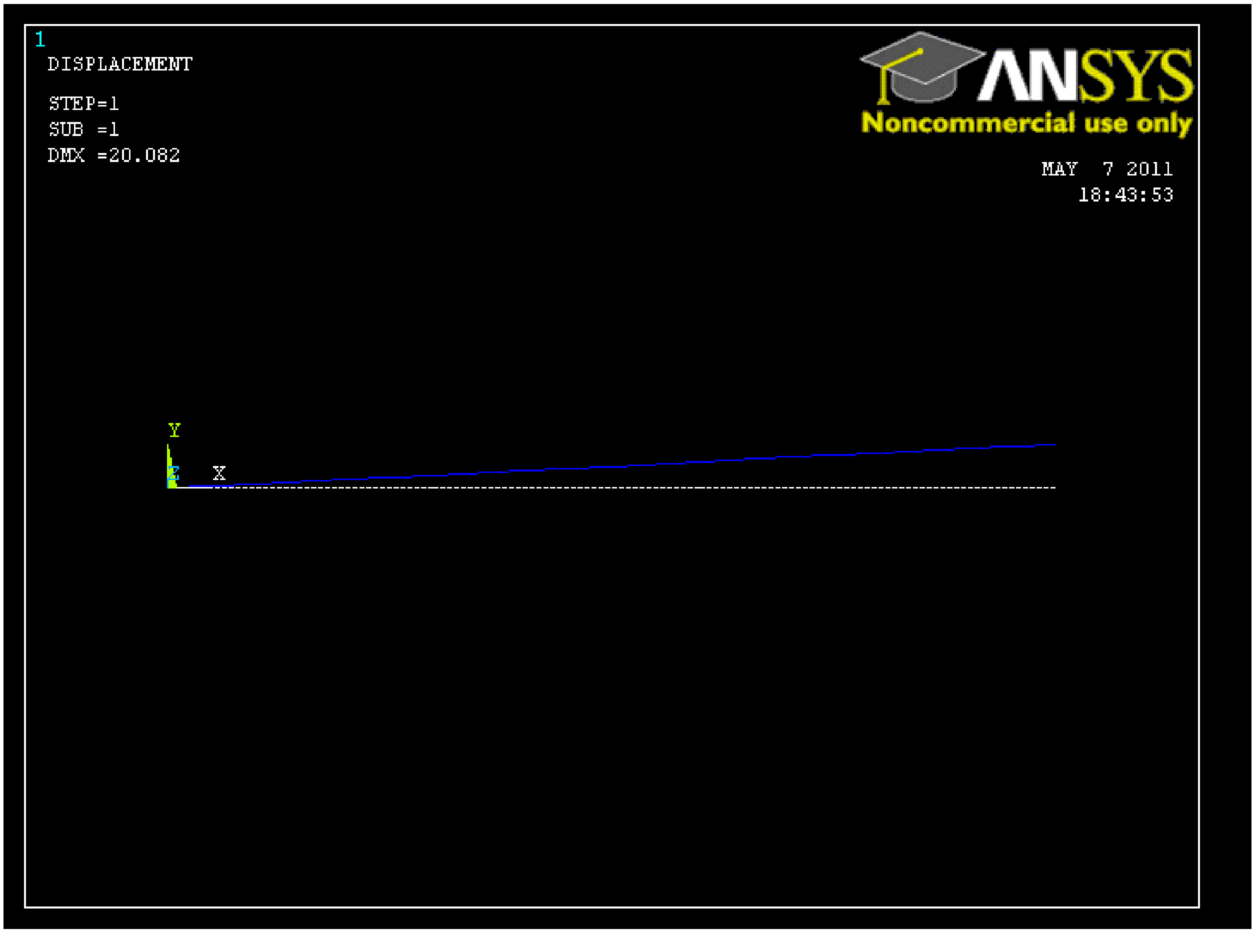
READ THE CALCULATED RESULTS OF THE SECOND AND THIRD MODES OF

VIBRATION

1.ANSYS Main Menu--General Postproc--Read Results--Next Set

2.Save_db

3. Exit. [9.1]

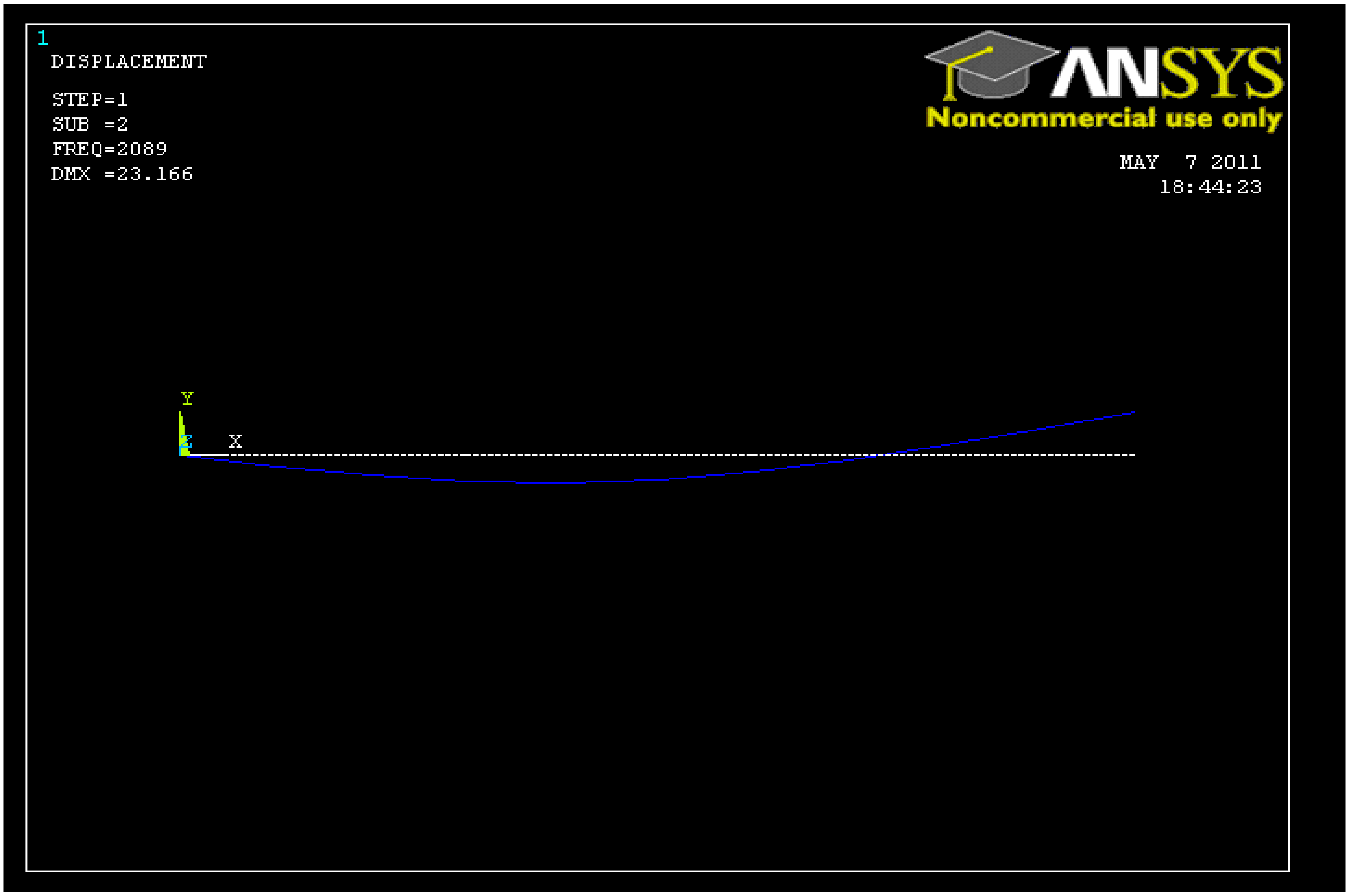


Clamped –Free (Step=1)

Step =1

Sub = 1

Maximum deformation = 20.082



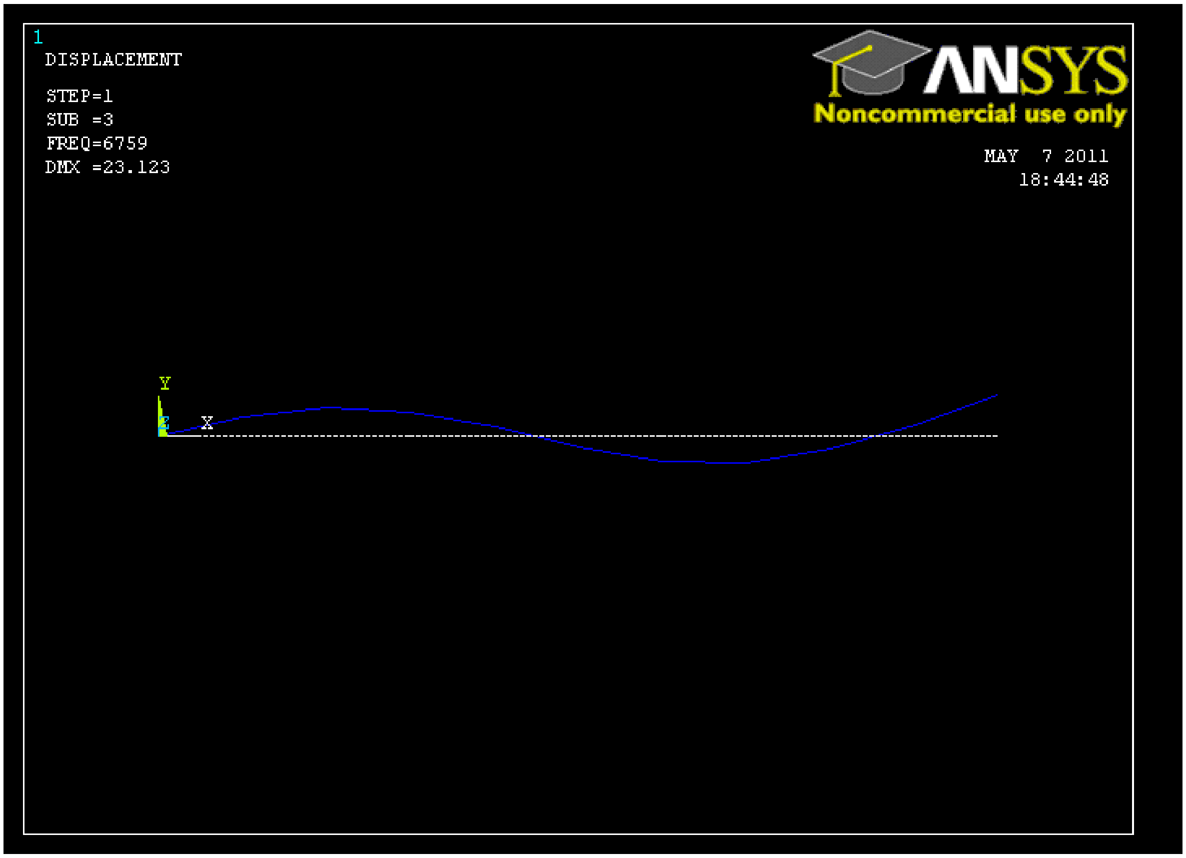
Clamped – Free (Step 2)

Step =1

Sub = 2

Maximum deformation = 23.166

Frequency =2089 Hrtz



Clamped- Free (Step 3)

Step =1

Sub = 1

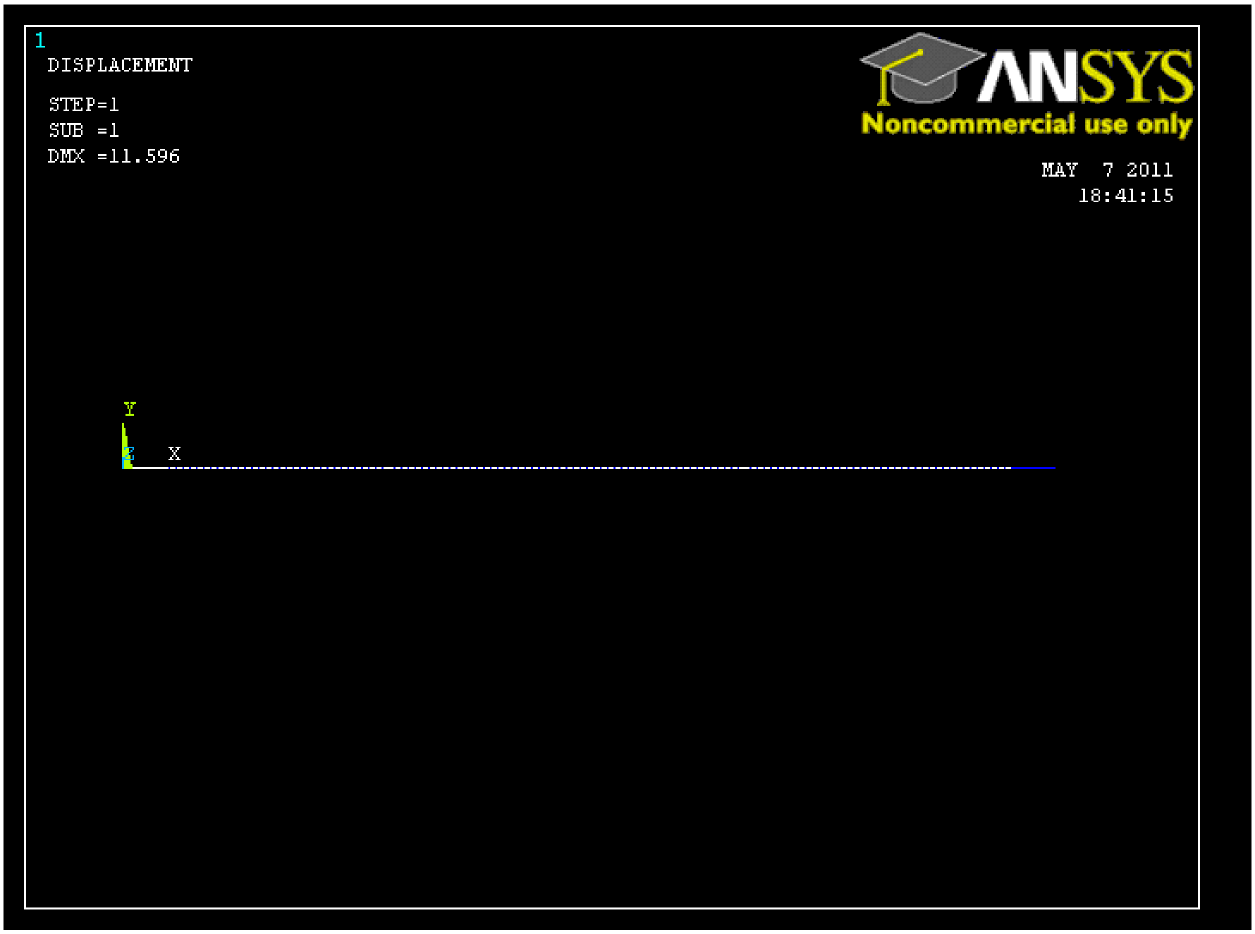
Maximum deformation = 23.123

Frequency =6759 Hrtz

Similarly analysis can be done for simple-simple supported graphite-epoxy beam :

For the sake of simplicity laminate is considered to be of unidirectional lamina and an additional force of 1000N is provided at middle of beam during analysis to make deflection visible .

Simple- Simple Supported Graphite- Epoxy Beam

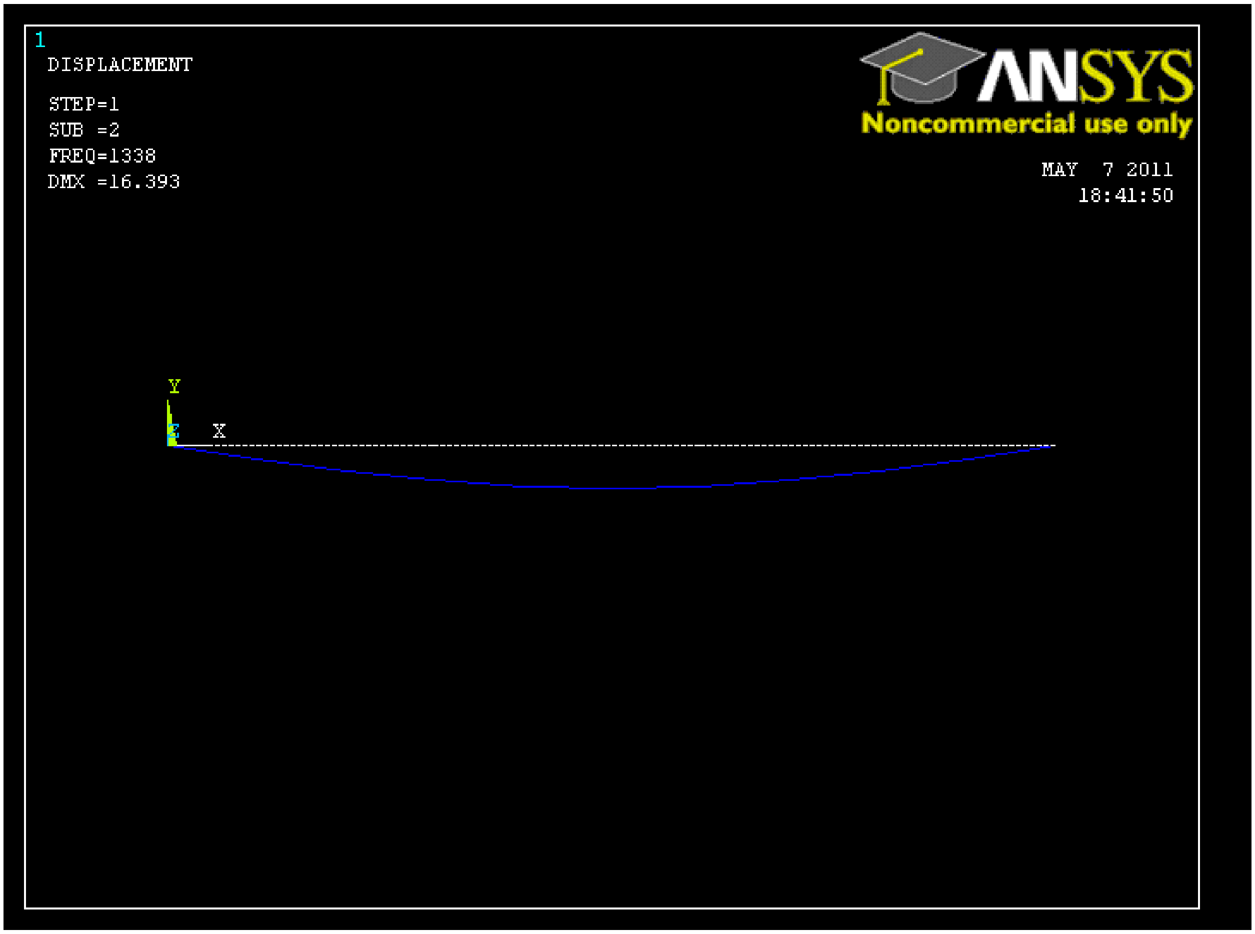


Step =1

Sub = 1

Maximum deformation = 11.596

Frequency -

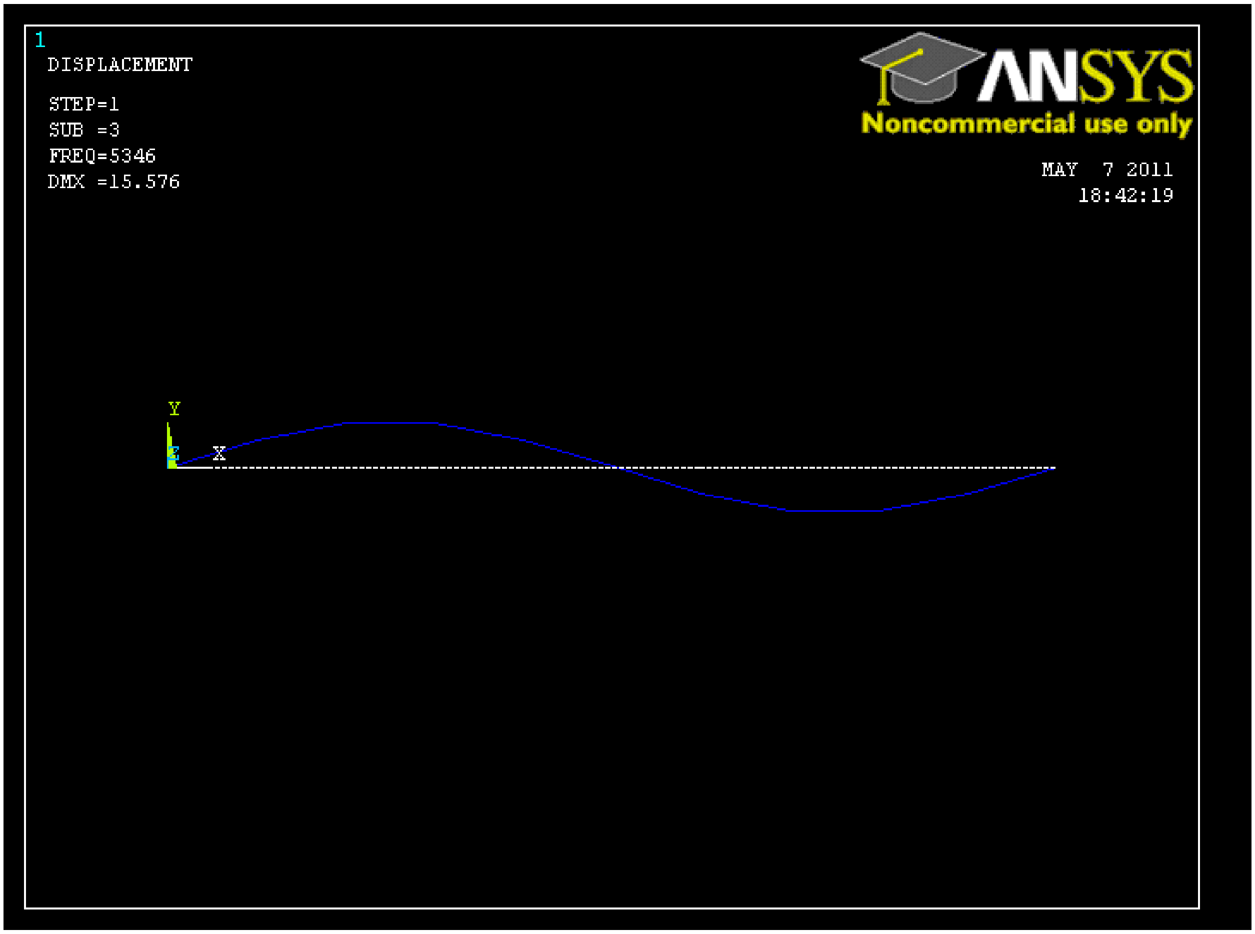


Step =1

Sub = 2

Maximum deformation = 16.393

Frequency =1338 Hrtz



Step =1

Sub = 1

Maximum deformation = 23.123

Frequency =5346Hrtz

CHAPTER -10

CONCLUSION

Natural frequency is obtained using classical beam theory for different types of laminated composites i.e Boron/Epoxy , Graphite/Epoxy and Glass/Epoxy . It was found that natural frequency increases with increase in mode of vibration . Natural frequency is minimum for clamped –free supported beam and maximum for clamped-clamped supported beam .In between these two, natural frequency of simple-simple and clamped-simple supported beam lies respectively . A comparative study between natural frequency of these composites was done for first mode of vibration for different types of support . It was found that Glass/Epoxy composite is having lowest value of natural frequency and Graphite / Epoxy composite is having highest value of natural frequency . Natural frequency of Boron/Epoxy composite is in between these two . Considering beam as Euler beam mode function was determined for different supporting conditions . Mode shape was plotted for differently supported beam with the help of matlab to get exact idea of mode shape . Vibrational analysis of beam was also done on Ansys to get natural frequency and same trend of natural frequency was found to be repeated .

CHAPTER – 11

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